

Dominant parameters for the critical tunneling current in bilayer exciton condensates

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We will discuss the relevant conditions to observe a critical tunneling current [New J. Phys. **10**, 045018 (2008)] in electron double-layer systems at a total filling factor of 1 and find they are related to the effective layer separation and the temperature. Our studies suggest that the intensity of the critical tunneling behavior is also directly linked to the area of the sample.

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I. INTRODUCTION

Under large perpendicular magnetic fields B the motion of charged carriers in two-dimensional electron systems (2DES) is confined to small cyclotron orbitals. This confinement suppresses the kinetic energy of the electrons but on the other hand amplifies their Coulomb interactions. In single layers these Coulomb correlations can lead to the emergence of fractional quantum Hall states. Two individual but closely spaced 2DES may also exhibit a correlated state, however, the underlying physics is now also influenced by the Coulomb interactions between the two systems. When the electron densities n in both layers are identical and the individual filling factors $\nu = nh/eB$ are close to $1/2$ (i.e., $\nu_{tot}=1$), the system may spontaneously develop interlayer phase coherence, provided the distance d between the layers is sufficiently small. This led to the prediction of Josephson-type phenomena in bilayer systems¹⁻³ nearly 20 years ago.⁴ The ratio of the center-to-center layer separation d and the magnetic length $l_B = \sqrt{\hbar/eB}$ is commonly used to parameterize the strength of this emerging state at the bilayer's total filling factor of 1. Tunneling spectroscopy experiments^{5,6} demonstrated the phase coherence between the two layers by showing a dramatic enhancement of the tunneling conductance at $\nu_{tot}=1$. More recently, a critical behavior was observed in dc tunneling experiments,⁷ which had also been predicted⁸ but failed to appear in all prior experiments. The purpose of this paper is to elucidate on the requirements to observe such a critical behavior. Our studies demonstrate that the coherent tunneling not only intensifies with the size of the sample but more importantly that the critical current grows linearly with the $\nu_{tot}=1$ area. In order to deliver a thorough picture of the critical tunneling behavior we first briefly address those experimental conditions, which can easily be manipulated, such as effective layer separation d/l_B , temperature, and filling factor, before discussing the more relevant size dependence.

II. EXPERIMENTAL DETAIL

Our data are obtained from samples of two different wafers grown in two different molecular-beam epitaxy (MBE) machines (hereby referred to as wafer α and β), both with a net barrier thickness of 9.6 nm, consisting of alternating AlAs (1.70 nm) and GaAs (0.28 nm) layers. Wafer α is the

same as used in Ref. 7. The intrinsic densities of approximately $4.0 \times 10^{10} \text{ cm}^{-2}$ (wafer α and β) of the two quantum wells originate from standard modulation doping. The low-temperature mobilities exceed $450000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ (wafer α) and $500000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ (wafer β). The single-particle tunnel splitting $\Delta_{S,AS}$ in our double quantum wells was estimated to be approximately $150 \text{ } \mu\text{K}$. Independent electrical contact to the two layers is achieved by growing the double quantum wells onto prestructured back gates⁹ and by using additional top gates in order to exploit a selective depletion technique.¹⁰ As the distance between back gate and 2DES is only around $1 \text{ } \mu\text{m}$, this overgrown back gate technique requires voltages of less than 1 V to locally deplete the contact arms and obtain independent electrical contacts to the two layers. The samples were either patterned into Hall bars of different sizes or a quasi-Corbino ring.¹¹ The specific sample dimensions will be given later in the text.

The dc I/V tunneling measurements presented in this paper are performed as sketched in Fig. 1(b), by applying a tunable dc bias voltage (hereinafter referred to as two-terminal voltage V_{2t}) between the two layers and detecting the current flow I toward ground as a voltage drop over a known resistance.⁷ As the interlayer phase coherence at $\nu_{tot}=1$ allows to easily transfer charges between the layers, the tunnel conductance becomes enormously enhanced which is tantamount to a very small tunnel resistance. Consequently, even if there is a finite bias V_{2t} applied, the interlayer voltage probes A and B (located close to source and drain) may read a value for V_{4t} of close to zero [Fig. 1(a)]. However, this coherent tunneling can be destroyed if the current [Fig. 1(c)] exceeds a critical value I_C . For the representative measurement presented in Fig. 1 the critical value is roughly $\pm 1.5 \text{ nA}$. Figure 1(d) is a depiction of the measured current I over the measured voltage V_{4t} . This representation will be used throughout this paper, and the critical currents $\pm I_C$ now translate into the maximal positive and negative currents at $V_{4t}=0$ as indicated by the dashed lines. The data here are presented in a scatter plot, which show a discontinuity in the measured current and voltage characteristics when the system moves from the coherent strong tunneling regime into the weak tunneling regime. The origin of this negative differential conductance is the sudden change in the total impedance $R_{tot}=R_T+R_S$, when a tunneling resistance R_T of almost zero is replaced by a resistance much larger than the series resistance R_S of the (uncorrelated) quantum Hall systems, i.e., $R_T \approx 0 \text{ } \Omega \rightarrow R_T \gg R_S$.

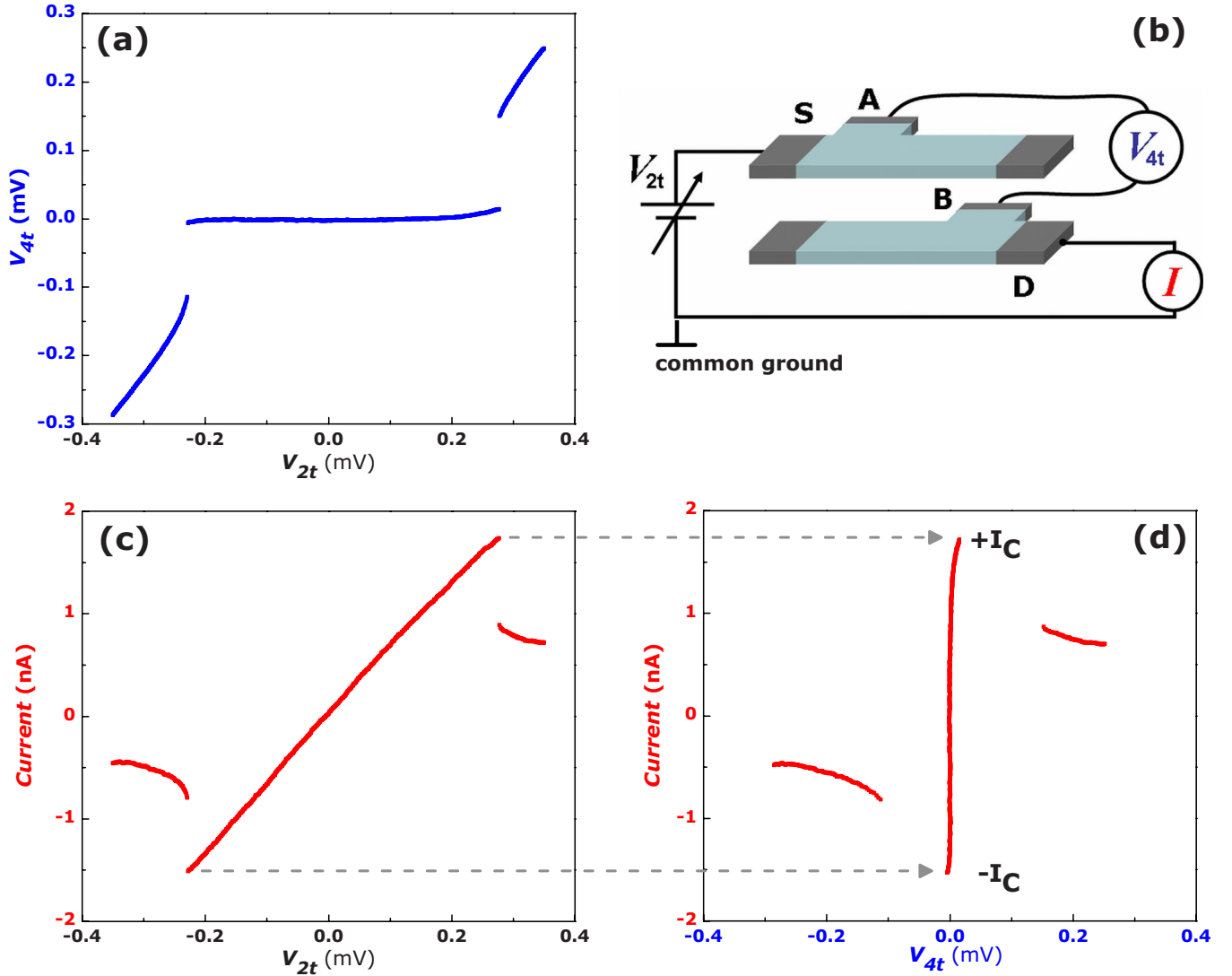


FIG. 1. (Color online) (a) Measured four-terminal voltage V_{4t} (c) and the measured total current I as a function of the applied two-terminal voltage V_{2t} , at $\nu_{tot}=1$ ($d/l_B=1.42$). When the current is plotted over V_{4t} , the curve (c) collapses onto subfigure (d). The absolute values of the negative and positive critical currents $\pm I_C$, however, remain unchanged and translate now into the maximal currents around $V_{4t}=0$. Subfigure (b) is a cartoon of the measurement setup with S and D being the source and drain contacts and A and B being the voltage probes. Hall bar sample of 0.88×0.08 mm² (wafer α) at $T < 20$ mK.

Historically, the study of coherent tunneling at $\nu_{tot}=1$ exploits the tunneling spectroscopy technique, where the differential tunneling conductance dI/dV is obtained in an ac measurement. These tunneling spectroscopy measurements (TSM) reveal a resonantly enhanced zero-bias tunneling peak at a total filling factor of 1.^{5,6,12–14} A critical behavior as discussed in this paper, however, is hidden in the dc part of the TSM, which is usually not shown. If the critical current is intrinsically small, it may also be difficult to detect or conceal through the influence of the ac modulation.

III. CRITICAL CURRENT VS FILLING FACTOR

It does not come as a surprise that the critical tunneling behavior (or the signatures of the $\nu_{tot}=1$ state in general) is strongest when the total filling factor of the system is exactly 1 or when the individual filling factors are exactly $1/2$,

respectively.¹⁵ Deviating to larger fields (smaller total filling factors) or smaller fields (larger total filling factors) will strongly suppress the coherent tunneling. In the uncorrelated regimes, carriers are exchanged between two individual two-dimensional electron systems, a process which now requires a finite amount of energy to overcome the Coulomb repulsion between the electrons. Figure 2(a) provides a qualitative picture of this behavior. It shows a set of several four-terminal I/V curves measured on a Hall bar sample of 0.88×0.08 mm² (wafer α) at $\nu_{tot}=1$ ($d/l_B=1.42$) and for small offsets in steps of about $\Delta\nu=0.3$ thereof. While drifting to either side of $\nu_{tot}=1$, the critical tunneling behavior is getting progressively suppressed around zero bias. Our data suggest that moving toward higher fields suppresses the tunneling slightly more rapidly than in the opposite low-field direction as the Coulomb exchange increases with $l_B^{-1} \propto B$. Figure 2(b) gives a quantitative analysis of the maximum (critical) cur-

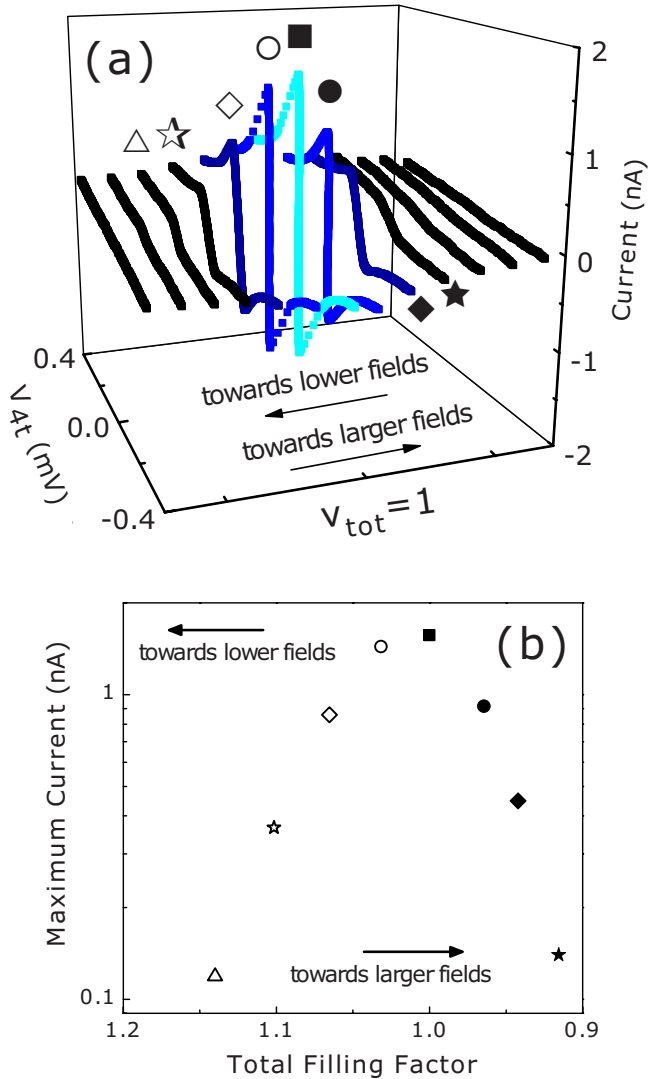


FIG. 2. (Color online) (a) Tunnel characteristics for several total filling factors, i.e., the electron density remains constant while I/V characteristics are measured at several magnetic fields around $\nu_{tot} = 1$. The eight most inner curves are marked with symbols. (b) Maximal (critical) currents as a function of the total filling factor [data from labeled curves in subfigure (a) are used]. At exactly $\nu_{tot} = 1$ the effective layer separation is 1.42. Hall bar sample of $0.88 \times 0.08 \text{ mm}^2$ (wafer α) and $T_{bath} < 20 \text{ mK}$.

rent as a function of the total filling factor. Note that only the eight plots in the center of Fig. 2(a) allowed the determination of a maximal current.

IV. CRITICAL CURRENT VS EFFECTIVE LAYER SEPARATION AND TEMPERATURE

As the total filling factor one state resides within a large parameter space, the size dependence we are about to discuss cannot be studied fully independent of other important parameters. The purpose of Sec. IV is thus to outline the effects of manipulating temperature and effective layer separation d/l_B , which largely influence the magnitude of the critical current as well. The latter is achieved experimentally by in-

creasing the electron densities in both layers simultaneously and adjusting the magnetic field, i.e., $B_{\nu_{tot}=1} \propto n_{tot}$. Figure 3(a) shows the positive and negative critical currents as a function of d/l_B for a Hall bar sample of $0.88 \times 0.08 \text{ mm}^2$ (wafer β) at below 20 mK. For $d/l_B \leq 1.3$ the current appears to saturate at 2 nA, even to decrease. This, however, is related to the effect of the sample's very low electron density, which begins to suppress the transport current altogether (the single-layer density is roughly $1.65 \times 10^{14} \text{ m}^{-2}$). For intermediate d/l_B on the other hand, the trend is clearly linear. For $d/l_B > 1.85$ the system undergoes a phase transition and the critical tunneling behavior (and the $\nu_{tot}=1$ QH state) disappears. This value is in very good agreement with those found in magnetotransport¹⁶ or tunneling spectroscopy experiments.⁵ Generally, if the onset of the $\nu_{tot}=1$ state is observed at $d/l_B \approx 2$, its origin is a pure many-body effect, and in weakly tunneling samples (i.e., $\Delta_{S,AS} \approx 0$) the phase coherence would develop spontaneously.¹⁶ The smooth phase transition we observed supports a “puddle model” as suggested by Stern and Halperin,¹⁷ where the $\nu_{tot}=1$ phase breaks up into domains near the phase boundary. In this model, the type of phase transition would have to be of first order as two phases coexist in the sample. Figure 3(c) shows three four-terminal I/V curves corresponding to a d/l_B of 1.40, 1.61, and 1.79.

The expected phase transition for correlated bilayers occurring at finite temperatures is not a regular second-order phase transition such as for normal superconductors at zero field or ferromagnets but a Kosterlitz-Thouless-type of phase transition. However, standard transport experiments are not able to judge the type and form of the occurring phase transition. Nevertheless, what we see experimentally in transport (i.e., tunneling) at finite temperatures is summarized in Fig. 3(b). There we plot the critical current as a function of the inverse temperature for a fixed $d/l_B = 1.42$ measured on a Hall bar sample of $0.88 \times 0.08 \text{ mm}^2$ (wafer α). Figure 3(d) shows several corresponding four-terminal I/V curves. At a temperature exceeding approximately 80 mK, the critical current begins to decrease rapidly. Extrapolation of the data in this region indicates the suppression of the critical behavior for temperatures above 250 mK. The $\nu_{tot}=1$ QH state as observed in magnetotransport is very weak at 250 mK and disappears entirely at temperatures exceeding approximately 350 mK.¹¹ The exact type of this phase transition is unknown to us. Spielman¹⁸ found similar overall trends on the temperature and d/l_B in tunneling spectroscopy measurements using samples, which display very small critical currents.

V. CRITICAL CURRENT VS SAMPLE SIZE

The following study of the size dependence was motivated by magnetotransport and tunneling measurements performed on a Corbino ring.¹¹ These experiments had shown a vanishing conductance across the annulus, suggesting that the bulk of the $\nu_{tot}=1$ phase is incompressible. As any inter-edge charge transfer is suppressed, tunneling would then (generally) only occur in the vicinity of the edges of the coherent $\nu_{tot}=1$ system and its magnitude would have to scale with the circumference of the sample. Tunneling spectroscopy measurements on Hall bar samples on the other

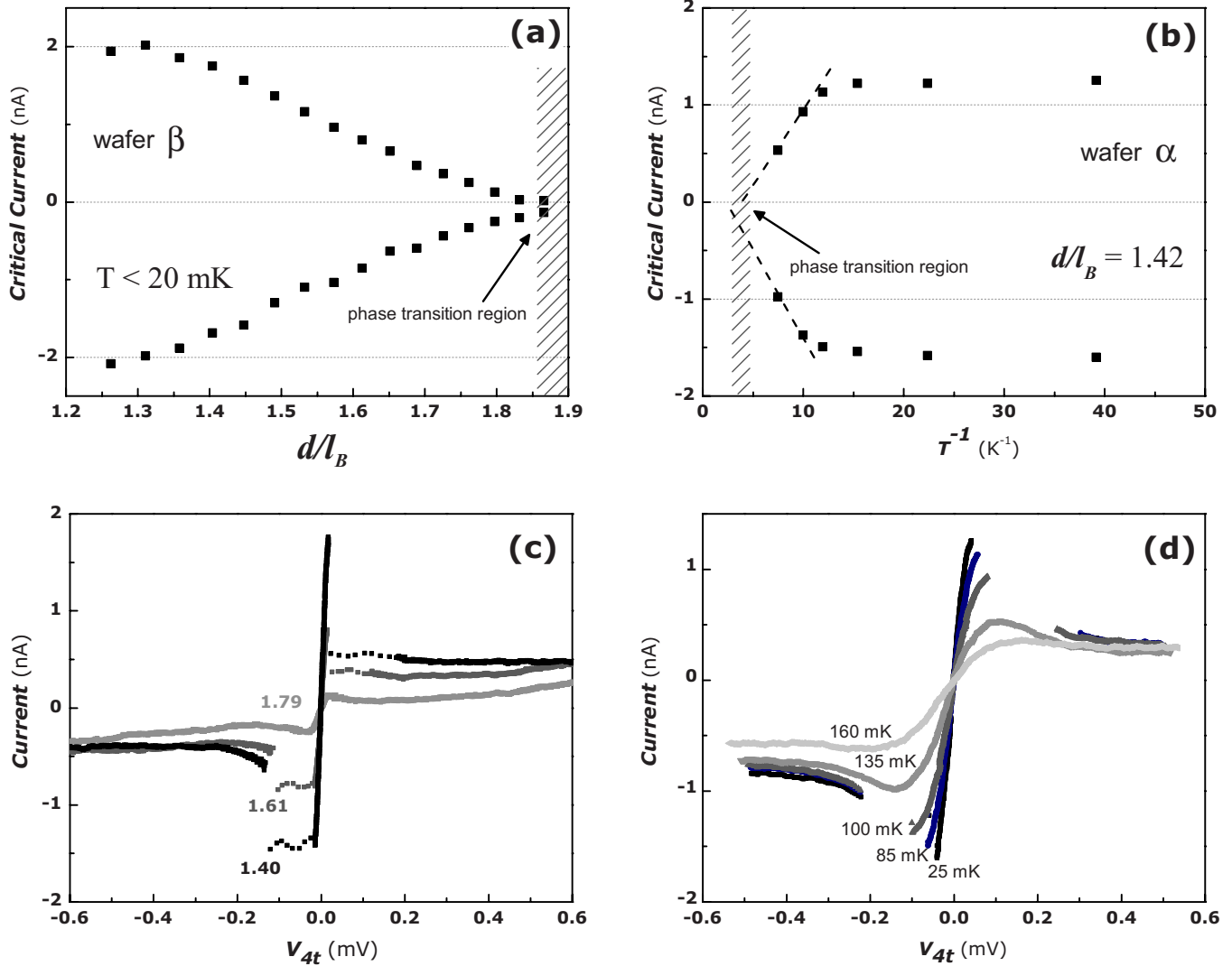


FIG. 3. (Color online) (a) Critical current as a function of the effective layer separation d/l_B at fixed temperature and (b) the inverse temperature at fixed effective layer separation. (c) and (d) illustrate some corresponding four-terminal I/V curves to allow comparison.

hand indicated that the zero-bias tunneling conductance may be related to the area of the sample instead.¹³ To obtain a better understanding and trying to solve this contradiction we compared the critical currents, rather the properties of the TSM tunneling peak, in terms of the circumference and area. Comparing different samples of course may introduce a systematic error yet we still found that the effect of different sample sizes had much more dramatic consequences as we will see next.

Figure 4(a) plots the value of the critical current as a function of the sample circumference U and as a function of the sample's $\nu_{tot}=1$ area A when the effective layer separation d/l_B for all samples is ≈ 1.6 . The data points are labeled with the wafer index α or β . Our data clearly indicate that the critical current I_C grows as the sample increases in size. When we apply a linear fit in the $\log\text{-}\log$ diagram, we find that the trend is best described by $1.04 \log(A)$ for the area and $2.11 \log(U)$ for the circumference. Based on these data we propose that the parameter that determines coherent tunneling at $\nu_{tot}=1$ is the sample area with a linear dependence and not the circumference.

Figure 4(b) shows the corresponding four-terminal I/V curves. Please note that the sample with the largest area (and circumference) displayed a strong hysteresis between up and down sweeping of the applied voltage V_{2t} . Figure 4(b) shows the upsweep, where the negative and positive critical currents differ, i.e., $I_C^- < I_C^+$. In a subsequent downsweep, the situation and the absolute values are reversed, i.e., $I_C^- < I_C^+$ and $I_C^+(\text{upsweep}) = I_C^-(\text{downsweep})$. For that reason, the fits in Fig. 4(a) ignore the error bar of these data points.

For comparison, we extracted a critical current of 17 pA at $d/l_B \approx 1.6$ from.^{6,18} This value originates from the dc part of a TSM on a $250 \times 250 \mu\text{m}^2$ sample. We believe that the disagreement between our data and this value is related to a different single-particle tunneling splitting $\Delta_{S,AS}$, which is determined by the height and width of the tunneling barrier, and to the large enhancement of the tunneling amplitude at $\nu_{tot}=1$.¹⁹ Their double quantum well is differently designed, with 18 nm GaAs quantum wells separated by a 9.9 nm barrier layer. The center-to-center separation d , however, is nearly identical to ours.

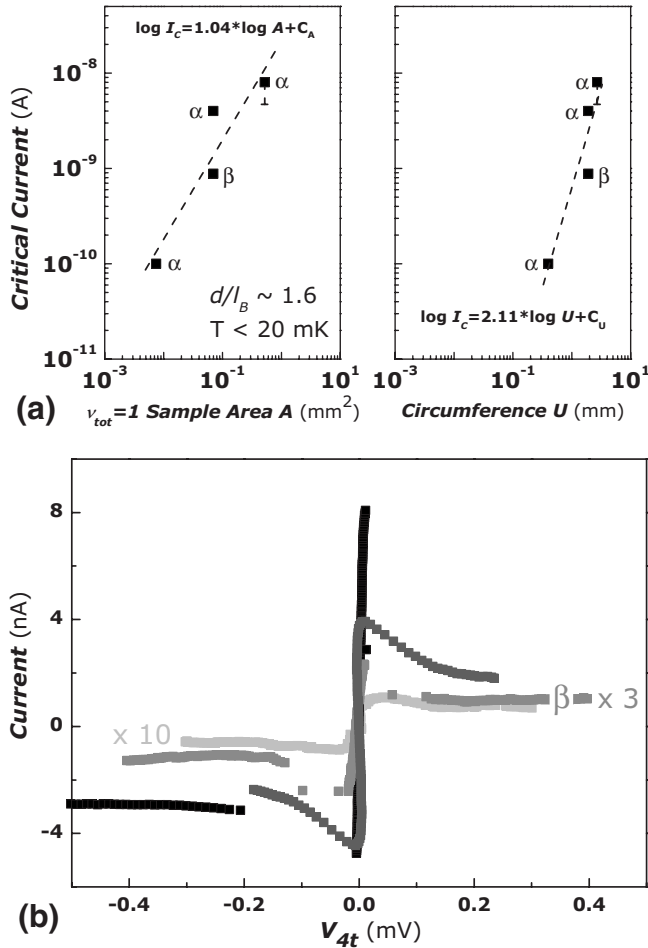


FIG. 4. (a) Critical currents as a function of the circumference U (right-hand side) and the $\nu_{tot}=1$ sample area A (left-hand side) for $d/l_B \approx 1.6$ for samples from wafer α and β . Fitting yields a linear dependence on the area. See text for further discussions. (b) Corresponding four-terminal I/V curves. The sample dimensions are (from largest to smallest): Corbino ring; $d_{outer}=860 \mu\text{m}$ and $d_{inner}=270 \mu\text{m}$, Hall bar type 1; $0.88 \times 0.08 \text{ mm}^2$, and Hall bar type 2; $0.15 \times 0.05 \text{ mm}^2$.

Our studies indicate that the initial notion of a length dependence on tunneling as mentioned at the beginning of this paper is a misinterpretation due to the astonishing properties of the coherent phase. Coherent interedge tunneling in a Corbino topology is suppressed because injected electrons do not become part of the $\nu_{tot}=1$ state. In the excitonic condensate picture this can be understood as it is impossible for the condensate to create an interlayer exciton by placing a hole at the site of the injected electron, when the drain lead where the hole has to originate from is far away on the other side of the bulk. True coherent tunneling on the other hand occurs when new interlayer excitons are created or existing ones are annihilated. These processes appear to be bulk phenomena after all. However, as the $\nu_{tot}=1$ phase could break up into domains near the phase boundary,¹⁷ it is possible that certain physical conditions may arise, which yield a different dependence on the area (and circumference) than the one presented here.

Generally, the observation of a critical behavior requires—vaguely speaking—“sufficiently large samples.”

The tangible sample size depends on the underlying design of the double quantum well and the sample structure, but may also be influenced by other unknown factors. More specifically, our studies identify the area rather than the circumference as the determining parameter for coherent tunneling at $\nu_{tot}=1$.

VI. DISCUSSION

A tunneling experiment in the general sense of charge transfer between two electron reservoirs through a sufficiently thin barrier is an inappropriate interpretation for the peculiar case of the total filling factor 1 state. Instead, as the two layers are considered to be indistinguishable, correlated interlayer tunneling is a direct signature of interlayer phase coherence. The model of indistinguishable layers also implies that the critical behavior and its dependence on the parameters we discussed in this paper have several implications for magnetotransport experiments performed in the regime of the total filling factor 1 state.²⁰ It can be shown that the leakage or tunneling current in the counterflow configuration^{21,22} depends on whether the driving current in the system is larger or smaller than the critical current for a given condition,²³ i.e., d/l_B , temperature and sample size. The observed gap in magnetotransport on the other hand is only slightly altered. It nevertheless implies that temperature-activation measurements at a total filling factor of 1 (Ref. 24) require a much more careful interpretation of the gap energy.

What makes matters experimentally complicated is the strong dependence of the critical current I_C on the size of the sample, meaning that for samples of certain dimensions the transport current may already be much larger than the critical current, even when the system is at base temperature. This fact could be able to account for several unsettled observations such as finite dissipation in counterflow experiments for example.²¹ Future magnetotransport experiments in the regime of the coherent total filling factor one state need to take the relevance of the driving current into account.

In summary, we discussed the relevant parameters necessary for the observation of a critical behavior in the coherent $\nu_{tot}=1$ state. We find a linear dependence of the value of the critical tunneling current I_C on the $\nu_{tot}=1$ area of the sample. I_C will decrease when the system is brought toward the phase boundary by increasing d/l_B or the temperature. It is also rapidly destroyed by moving away from a total filling factor of 1.

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